

Tangram and Beyond (2)

by Fred Horn

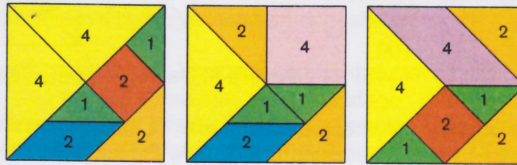


Figure 1. Solutions based on the use of 16 triangles with *Tangram* on the left

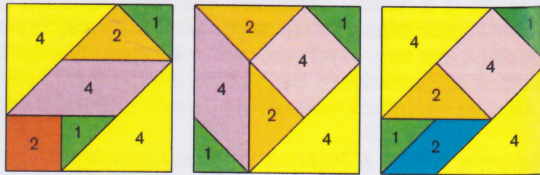


Figure 2. Solutions based on the use of 18 triangles

Tangram consists of five triangles, a square and a parallelogram. Together the seven pieces form a square. Each piece is made up of a number of smaller triangles. Are there variations possible to the traditional sizes of these seven pieces? This was the question posed originally in [1] and again in the previous article [2] of this series.

At the time, only a single person submitted the right answer of six possibilities, which he derived based on the areas of the pieces and the fact that their sum should equal the area of the square. A more elegant explanation can be based on the smallest triangle. The other pieces are named on the basis of the number of these triangles they contain. Each collection of the five triangles consists of

- 1, 2, 2, 4, 4 = 13,
- 1, 1, 2, 4, 4 = 12, or
- 1, 1, 2, 2, 4 = 10

of these small triangles. They are supplemented with a square and a parallelogram, either of which can consist of two, four or eight small triangles. This leads to the summary in Table 1 of the various combinations of seven pieces with their corresponding number of small triangles.

Five triangles	Parallelogram or square					
	2, 2	2, 4	4, 4	2, 8	4, 8	8, 8
1, 2, 2, 4, 4	17	19	21	23	25	29
1, 1, 2, 4, 4	16	18	20	22	24	28
1, 1, 2, 2, 4	14	16	18	20	22	26

Table 1. Possible number of small triangles

The square in which the seven pieces must fit will also have to be build up by these numbers of triangles. Thereby basic units of either two or four triangles must be used as shown in Figure 3. On the basis of the number of triangles at the edge of the square, the number of small triangles in squares whose sizes increase consecutively can be:

- a) a single triangle: 2, 4
- b) two triangles: 8, 16
- c) three triangles: 18, 36
- d) four triangles: 32, 64
- e) five triangles: 50, 100
- f) six triangles: 72, 144

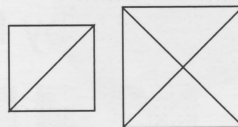


Figure 3. Basic units made from triangles

The only numbers occurring in these sets and also in Table 1 are 16 and 18, which lead to six theoretical possibilities among which is the original *Tangram*. They are shown in Figure 1 and 2.

The follow-up question is a natural extension: In how many ways can we make a square out of these six possibilities of the seven pieces if rotation is excluded? The original *Tangram* allows for two, which are mirror images. The full answer will be given in the next article in this series.

References

- [1] Fred Horn, *Tangram*, Natuur & Techniek Wetenschapsmagazine, number 6 (June 2002), p. 78.
- [2] Fred Horn, *Tangram and Beyond*, CFF 82 (2010), p. 14.