

The MacMahon Cubes

by Fred Horn

Ask anybody how many different six-sided dice are possible and they will look at you as if you have lost your mind. "Endless of course" they will answer, referring to size, material, colour, symbols, etc. And when you then add: "every pairs of opposite sides add up to seven!". A smart person might then answer with: "Two, the clockwise numbered die and its mirror image", see Figure 1.



Figure 1. A die and its mirror image



Figure 2. Arrangements of pips

But when we also consider the arrangements of the pips denoting the numbers on the sides, it is clear that more solutions are possible. Three pip arrangements: 1, 4, and 5 have fourfold symmetry. The other three pip arrangements: 2, 3, and 6 have twofold symmetry, see Figure 2. When we combine this under the condition that opposite sides add up to a total of seven pips, we find $2 \times 2 \times 2 \times 2 = 16$ different dice.

Can a similar analysis can be made for the face-coloured cubes used for a lot of children's games? As there is no standard for the order of the colours, the question is how many different cubes are possible? At the end of the nineteenth century Percy Alexander MacMahon, major in the English Army, asked himself the same question: "When I use six colours and I colour the faces of a six-sided cube different using these colours, how many different cubes will that produce?" There are exactly thirty different ways to colour the cubes under these conditions. In 1892 major MacMahon registered *MAYBLOX* with eight cubes out of the total set of thirty. *MAYBLOX*, see Figure 3, was produced and published by R. Journet from London in 1892. Goal of the puzzle is to assemble a $2 \times 2 \times 2$ cube with all six colours on the outside and the same colour on touching sides inside, see Figure 4.

In his book *New Mathematical Pastimes* [1] from 1921 major MacMahon describes a more complicated puzzle that can be played with the thirty cubes. The idea is to take one cube out of the set of thirty and use this as an example-cube. Then find the eight cubes out of the remaining twenty-nine that can form a $2 \times 2 \times 2$ cube conform



Figure 3. *MAYBLOX*

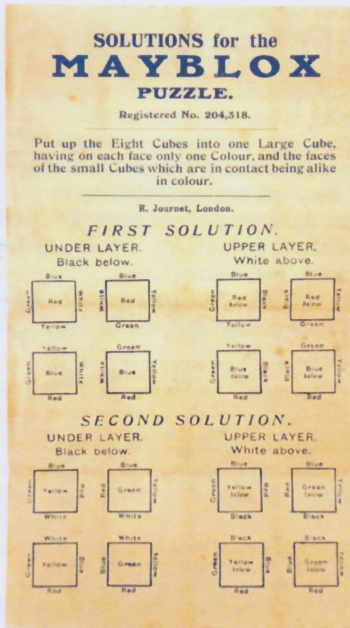
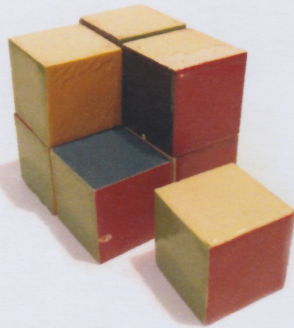


Figure 4. MAYBLOX solution sheet and first solution



the rules of *MAYBLOX* with the outside colour scheme of the example-cube. Then major MacMahon shows that from the remaining twenty-one cubes also the mirror image of the example cube can be build. In fact, it is said that from the remaining twenty-one cubes the player can again choose one cube as an example-cube and find eight others to make the bigger one!

Today it is not so easy to find a set of MacMahon cubes, but it is easy to manufacture a set yourself. Buy as many sets of block-

puzzles for children as you need to get thirty cubes. You can of course also use clean wooden cubes, but painting will be difficult because of the porosity of the wood. The block-puzzle cubes have a paper outside, which make them easy to paint. You need some patience and paint in six contrasting colours (for example red, blue, yellow, green, orange and purple) to create 30 different cubes. See Figure 5 for my own self-painted set, I used black and white instead of orange and purple.



Figure 5. Set of thirty different cubes, painted by Fred Horn

References:

- [1] Major P.A. MacMahon, R.A., *New Mathematical Pastimes*, Cambridge University Press, 1921, pp 42-47.
- [2] <http://www.robspuzzlepage.com/pattern.htm#macmahon>.