The development of a

TANGRAM family

Article by Fred Horn

Mathematically the TANGRAM is a simple puzzle consisting of seven pieces which can be combined into a square.

Figure 1

INTRODUCTION

It is 1976, and with my good friend Johan Sieders I am discussing if there is more, in a mathematical sense, about Tangrams than what was recently published in Joost Eiffers' book: “DuMont's Kopf-Zerbrecher” (ISBN 3-7701-0731-4) in the chapter “Classification and counting” by Michel Dekking with support by Jaap Goudsmit. They defined in three sub-chapters—1. Convex Tangrams; 2. Grid Tangrams; and 3. Split-up Tangrams (text between pages 123 and 124)—a lot of possibilities on what to do with the ‘normal’ 7-piece Tangram as described in the book.

Our discussions pushed us completely in an alternative direction. Looking at the different pieces, out of which a Tangram is composed, we thought about a square that had to be filled up with pieces of the same shape but a different size. So we decided to try to find some extensions of this well-known 7-piece Tangram.

How this worked out was published in 2002 in the magazine “Natuurwetenschap en Techniek” in a series of articles in Dutch. The English version of this series was published in the magazine “CFF” (Issues 82; 83; 84; and 85) of the Dutch Cube Club in 2010.

The first chapter of this article is a reworked version of these articles. In the second chapter I will introduce the ‘complete family.’

CHAPTER 1

The TANGRAM created a craze in the western world four times in the last two centuries. The history of the Tangram is extensively discussed in "The Tangram Book" by Jerry Slocum (ISBN 1-4027-0413-5) so I will refer to that book and not elaborate on that here.

Mathematically the Tangram is a simple puzzle. It consists of seven pieces which can be combined into a square as shown in Figure 1.

The seven pieces are all derived from a basic unit, which is a right-angled isosceles triangle. The original Tangram consisted of two of these basic triangles; the three possible ways to create a piece of double the area (a triangle; a square; a parallelogram) and twice the triangle with its area quadrupled. These pieces can be combined into a square in only two ways, which are mirror solutions, when rotations are ignored. The ‘actual’ puzzle consists of recreating “given pictures of pre-set shapes” with all seven pieces.

"The Tangram Book" by Jerry Slocum, 2003
But what if some of the mathematical conditions for the creation of the pieces are loosened?

— It should remain a puzzle consisting of seven pieces that fit into a square.

— It should still contain five triangles, but not necessarily made out of and ordered in increasing size; 1; 1; 2; 4; 4 basic triangles.

— Furthermore, both the parallelogram and the square may be a doubled version of the size 2- or size 4-triangles.

So our question was: Is this possible, and if so, how many different variants exist?

To solve the problem, we started with the square where the pieces had to fit in.

The square had to be constructed out of basic triangles. Therefore, basic units of either 2 (a) or 4 (b) triangles must be used, units as shown in Figure 2.

On the basis of the number of triangles at the edges of the square, the number of small triangles in, consecutively in size, increasing squares is as follows:

\[ a = 2; 8; 18; 32; 50; 72 \]
\[ b = 4; 16; 36; 64; 100; 144 \]

The next step was to look at the area needed for the possible combinations of the seven pieces. Starting with the triangle possibilities and their required area in basic triangle’s (bt):

I) \[ 1; 2; 2; 4; 4 \] gives 13 bt
II) \[ 1; 1; 2; 4; 4 \] ” 12 bt
III) \[ 1; 1; 2; 2; 4 \] ” 10 bt

These must be supplemented with a square and a parallelogram either of which can consist of 2 bt, 4 bt, or 8 bt. This leads to the summary in bt of the various combinations of seven pieces as shown in Figure 3.

The only numbers occurring in the ‘needed area’ and in the table of Figure 3 are 16 and 18, which leads to six theoretical possibilities among which is the original Tangram.

The three possibilities for a square of 16 bt are shown in Figure 4.

The three possibilities for a square of 18 bt are shown in Figure 5.

The follow-up question is a natural extension: In how many ways can a square be made out of these six possibilities of seven pieces if rotation is excluded?

The original Tangram allows for two, which are mirror images. For this original one and for the other five, all possible fill-ins (no rotation!) are pictured below in Figure 6.
Numerologists and others who look for some kind of meaning behind ordinary sets of numbers will have a field day with the solutions found!

All the squares that consist of 16 bt (rows 1-3 on preceding page) have exactly 16 different placements, and all of the squares that consist of 18 bt (rows 4-6) have exactly 18 as well!

In reality, in order to realize all of the above found placements, a set of Tangram puzzle pieces would have to be made containing the shapes from Figure 7.

But what if this condition (to only fit in a larger square) is loosened a bit by also allowing a triangular Tangram? Does in that case, referring back to Figure 3, present a total number of bts to make such a shape possible?

A triangle can be formed from a square when its area is halved.

Hence, the only option is the set of 25 bt as that is half the area of a square ‘5 by 5’.

<table>
<thead>
<tr>
<th>Parallelogram or square</th>
<th>Units 1,2,2,4,4</th>
<th>2,8</th>
<th>4,8</th>
<th>8,8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Five triangles</td>
<td>23</td>
<td>25</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>1,1,2,4,4</td>
<td>22</td>
<td>24</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>1,1,2,2,4</td>
<td>20</td>
<td>22</td>
<td>26</td>
<td></td>
</tr>
</tbody>
</table>

Is it possible to combine in such a triangle the seven pieces including one 8-bt piece? Yes!

The extra pieces tally a sum of 42 bt (26 + 16) which allow for a rectangular box of 3 x 7 small squares of twice the size of a basic triangle, and ‘low and behold,’ for all 12 pieces to fit in.
I will end Chapter 1 with another idea that came out of the discussions with my friend Johan Sieders.

The original Tangram is also a composition where one of the pieces that has to fit in has the same form as the outside square. Is it possible to make more Tangrams like that?

Looking at the outside form I thought of solutions with a triangle; square; pentagon; and a hexagon, all being formed with that small piece and a total of seven pieces.

The triangle and the square are described above, but here are my solutions for the other two:

Pentagonal Tangram

Hexagonal Tangram

The Tangram of Figure 12 has only remained in my files as a picture, but the Hexagonal Tangram example was produced twice and named HEXTAN (© Fred Horn 11/1987).

For the first production, the ICI-Tangram, see:

http://www.hong.nl/index.asp?z=ICI%2060%20jaar%20%20%20januar%201988&vi=1

For its second production, HEXTAN was fashioned as “Winter Glow Tangram” by NovaCarta in 2009.

Now possessing the complete set, the next anticipated question to be solved was how many solutions were possible — when returning to the square as the item to be filled in — when seven random pieces are selected out of this box?

Again, the large square could only have an area of 16 or 18 bt, so this would be the start for which combinations would be possible. Also the pieces utilized must have a size of 1 bt, 2 bt, or 4 bt. The use of the two largest 8 bt pieces proves impossible because a) they do not fit in (the parallelogram) or b) they leave no room for the other six pieces (the square).

N.B.*: With the 18 bt square, theoretically one solution is possible: 1; 1; 2; 2; 2; 2; 4. This one is ignored, although it can be laid down.

Another constraint is that only two 1 bt pieces, four 2 bt pieces, and four 4 bt pieces are in the box. With that in mind, the combination: 2; 2; 2; 2; 2; 2; 4 for the 16 bt square is not an option.

For the 16 bt square only one combination is possible: 1; 1; 2; 2; 4; 4.

Also, for the 18 bt square, only one combination is possible: 1; 1; 2; 2; 4; 4.
The 16 bt Tangram with its possible pieces:

Besides the two 1 bt triangles that must be used for all of the solutions, the following other pieces are necessary:

A) Two 2 bt triangles; one 2 bt square; two 4 bt triangles.
B) Two 2 bt triangles; one 2 bt square; two 4 bt triangles; one 4 bt triangle; one 4 bt square.
C) Two 2 bt triangles; one 2 bt square; one 4 bt triangle; one 4 bt parallelogram.
D) Two 2 bt triangles; one 2 bt square; one 4 bt square; one 4 bt parallelogram.
E) Two 2 bt triangles; one 2 bt parallelogram; two 4 bt triangles.
F) Two 2 bt triangles; one 2 bt parallelogram; one 4 bt triangle; one 4 bt square.
G) Two 2 bt triangles; one 2 bt parallelogram; one 4 bt triangle; one 4 bt parallelogram.
H) Two 2 bt triangles; one 2 bt parallelogram; one 4 bt square; one 4 bt parallelogram.
I) One 2 bt triangle; one 2 bt square; one 2 bt parallelogram; two 4 bt triangles.
J) One 2 bt triangle; one 2 bt square; one 2 bt parallelogram; one 4 bt triangle; one 4 bt square.
K) One 2 bt triangle; one 2 bt square; one 2 bt parallelogram; one 4 bt triangle; one 4 bt parallelogram.
L) One 2 bt triangle; one 2 bt square; one 2 bt parallelogram; one 4 bt square; one 4 bt parallelogram.

The 18 bt Tangram with its possible pieces:

Besides the two 1 bt triangles that must be used for all of the solutions, the following other pieces are necessary:

A) Two 2 bt triangles; two 4 bt triangles; one 4 bt square.
B) Two 2 bt triangles; two 4 bt triangles; one 4 bt parallelogram.
C) Two 2 bt triangles; one 4 bt triangle; one 4 bt parallelogram; one 4 bt square.
D) One 2 bt triangle; one 2 bt square; two 4 bt triangles; one 4 bt square.
E) One 2 bt triangle; one 2 bt square; two 4 bt triangles; one 4 bt parallelogram.
F) One 2 bt triangle; one 2 bt square; one 4 bt triangle; one 4 bt square.
G) One 2 bt triangle; one 2 bt parallelogram; two 4 bt triangles; one 4 bt square.
H) One 2 bt triangle; one 2 bt parallelogram; two 4 bt triangles; one 4 bt square.
I) One 2 bt triangle; one 2 bt parallelogram; one 4 bt triangle; one 4 bt parallelogram.
J) One 2 bt triangle; one 2 bt parallelogram; two 4 bt triangles; one 4 bt square.
K) One 2 bt square; one 2 bt parallelogram; two 4 bt triangles; one 4 bt parallelogram.
L) One 2 bt square; one 2 bt parallelogram; one 4 bt triangle; one 4 bt parallelogram; one 4 bt square.

Theoretically, 24 different Tangrams could be made with the above given distribution of pieces. And of course, all six Tangrams earlier described would be part of that:

1 = 16-I; 2 = 16-F; 3 = 16-C; 4 = 18-E; 5 = 18-C; 6 = 18-G.

But "the proof of the pudding is in the eating!" thus remains a last question: Does each other set of pieces fit into their larger square?

Some do and some don't!

For the 16 bt square, the combinations 16-J; 16-K; and 16-L do not fit.
For the 18 bt square the combinations 18-F; 18-I; 18-J; and 18-L do not fit.

But here are the combinations that do fit into a square:

16A 16B 16D
16E 16G 16H
18A 18B 18D
18H 18K

But still there are also a few more curious solutions. The theoretical *18 bt square solution looks like:

There are also two (the third one does not fit! solutions with the two 8 bt pieces:

32A = 2x 16D 32B = 2x 16H
This “Family of Tangrams” is presented here for the first time.

The ‘normal’ Tangram can be found everywhere. I only have to refer to Jerry’s before-mentioned book for the full story and a lot of examples.

There is also one other Tangram work that had been published before. In 1891, the firm F. Ad. Richter & Co., producing the ‘Anchor Puzzle problems,’ started with a new design in this series: PYTHAGORAS. The same design was, in 1913, used for their puzzle “Picollo Nr. 2.” See ‘The Anchor Puzzle Book’ (ISBN 978-1-890980-09-2).

For me, the most elegant member of this “Family” is the Tangram shown at right (Figure 15), but I do love all the others too!

---

**YOUR AGPI PLATINUM MEMBERSHIP BENEFITS**

United States ($40 per year);
Canada ($50 per year); Other countries ($70 per year)

- 4 issues of our print magazine *AGPI Quarterly*, sent by First-Class Mail. This publication features game and puzzle-related news, in-depth research information, as well as classified advertising.
- 6 issues of our online newsletter *On the Table*
- Invitation to the annual AGPI Convention (Guest speakers, sales and swaps, museum visits, play, auctions, and more)
- Invitation to Regional Meetings
- Full membership voting rights
- Full access to our website, www.gamesandpuzzles.org with extensive content (past publications, copies of historic game company catalogs, our comprehensive database with 18,000 game listings, and more.)
- Online forums and blogs on the website
- **Member Handbook & Directory**—provides a confidential member list, including their collecting and research interests.

Don’t forget!
Back issues of our Quarterly publications are still available.
Do you have them all?

Contact our archivist, Lisa Bloome, at toystoyou@aol.com

WWW.GAMESANDPUZZLES.ORG • SPRING 2017 13